

Section 6.1 Problems

Problem 1. Use the Gaussian Elimination Algorithm to solve the following linear systems, if possibly, and determine whether row interchanges are necessary:

$$\begin{cases} x - y + 3z & = 2 \\ 3x - 3y + z & = 1 \\ x + y & = 3 \end{cases}$$

Problem 2. Given the linear system

$$\begin{cases} x - y + \alpha z & = -2 \\ -x + 2y - \alpha z & = 3 \\ \alpha x + y + z & = 2 \end{cases}$$

1. find the values of α for which the system has no solutions.
2. find the values of α for which the system has an infinite number of solutions.
3. assuming a unique solution exists for a given α , find the solution.

Section 6.2 Problems

Problem 3. Find the row interchanges that are required to solve the following linear system using Algorithm 6.1

$$\begin{cases} x - 5y + z & = 7 \\ 10x + 20z & = 6 \\ 5x - z & = 4 \end{cases}$$

Section 6.3 Problems

Problem 4. Is the following matrix nonsingular? If it is compute the inverse:

$$\begin{pmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{pmatrix}$$

Problem 5. Prove the following:

1. if A^{-1} exists, it is unique
2. if A is nonsingular, then $(A^{-1})^{-1} = A$
3. If A and B are nonsingular matrices of the same size, then $(AB)^{-1} = B^{-1}A^{-1}$